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RADIATION FROM TWO PARALLEL CYLINDERS IN A VISCOUS MEDIUM

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ABSTRACT. Questions dealing with the theory of the radiation and propagation of waves from the surfaces of two parallel circular cylinders in a viscous medium obeying linearized Navier-Stokes equations are discussed.

#1. The propagation of sound waves radiated from the surface of a /58* circular cylinder in a perfect fluid has been investigated in detail, as in [3], for example. The theory of radiation from a cylinder in a viscous medium was the subject of article [4]. What follows will take up some of the questions of the theory of the radiation and propagation of waves from the surfaces of two parallel circular cylinders in a viscous medium obeying linearized Navier-Stokes equations.

If the fluid velocity is represented by the sum of two vectors

$$V = (\operatorname{grad}\varphi + \operatorname{rot}\psi)\, e^{-i\omega t}, \tag{1.1}$$

the solution of the linearized Navier-Stokes equations can be reduced to the solution of two wave equations [2] for the functions φ and ψ

$$\nabla^2\varphi + \alpha^2\varphi = 0;\ \alpha^2 = \frac{\omega^2}{c^2}\,\frac{1 + i\omega c^{-2}\left(\nu' + \frac{4}{3}\nu\right)}{1 + \omega^2 c^{-4}\left(\nu' + \frac{4}{3}\nu\right)^2};$$
$$\nabla^2\vec{\psi} + \beta^2\vec{\psi} = 0;\quad \beta^2 = i\omega\nu^{-1}. \tag{1.2}$$

Here ∇^2 is the Laplace operator; ω is the angular frequency of the oscillations; c is the speed of sound in an undisturbed fluid; ν' and ν are the kinematic viscosities.

Let us note that Equation (1.2) is descriptive only of small disturbances of a viscous fluid.

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* Numbers in the margin indicate pagination in the foreign text.

Fluid pressure p, and compression s, are linked with the function φ by the formulas

$$p = \varrho_0 \left[i\omega\varphi + \left(\nu' + \frac{4}{3}\nu \right) \nabla^2\varphi \right] e^{-i\omega t}; \quad s = -\frac{i}{\omega} \nabla^2 \varphi e^{-i\omega t}, \tag{1.3}$$

where

ϱ_o is the density of the fluid in the undisturbed condition.

Let us introduce two systems of cylindrical coordinates r_1, θ_1, z_1 and r_2, θ_2, z_2 (Fig. 1). The z_1 and z_2 axes are unalterably connected to the axes of the cylinders, the surfaces of which are the sources of the propagation of the sound waves. The cylinders are assumed to have one radius, and are in an unbounded fluid. The velocity of points on the surface of the cylinders can be arbitrary, but does not depend on the z coordinates and can be represented by a Fourier series. The boundary conditions on the surfaces of the cylinder will be /59

$$v_r = e^{-i\omega t} \sum_{n=0}^{\infty} V_n \cos n\theta, \quad v_\theta = 0 \quad \text{при} \quad r = a \tag{1.4}$$

when r = a, (a is the radius of the cylinder).

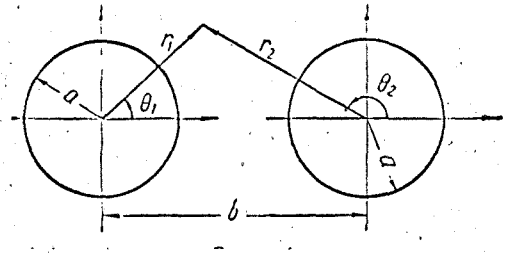

Figure 1

Let us note that in this case $\vec{\psi} = \vec{k}\psi$, where $\vec{k}$ is the unit vector along the z axis.

#2. Let us seek the solution of the equations at (1.2) in the form of the sum of the two functions

$$\varphi = \varphi_1 + \varphi_2; \quad \psi = \psi_1 + \psi_2, \tag{2.1}$$

each of which is a solution of the corresponding equation in a "local" system of coordinates, and which, in the general case and with the conditions for infinity taken into consideration, can be represented by the formulas

$$\varphi_i = \sum_{n=0}^{\infty} A_n^{(i)} H_n^{(1)}(\alpha r_i) \cos n\theta_i;$$
$$\psi_i = \sum_{n=1}^{\infty} B_n^{(i)} H_n^{(1)}(\beta r_i) \sin n\theta_i. \tag{2.2}$$

Here $A_n^{(i)}$, $B_n^{(i)}$ are arbitrary constants; $H_n^{(1)}(x)$ is a Hankel function of the first kind. The superscript of the Hankel function will be omitted henceforth.

According to Equation (1.1), the components of the velocity of the fluid particles can be determined through the expressions

$$v_r = \mathrm{Re}\, e^{-i\omega t}\left[\frac{\partial(\varphi_1 + \varphi_2)}{\partial r_i} + \frac{1}{r_i}\frac{\partial(\psi_1 + \psi_2)}{\partial \theta_i}\right];$$
$$v_\theta = \mathrm{Re}\, e^{-i\omega t}\left[\frac{1}{r_i}\frac{\partial(\varphi_1 + \varphi_2)}{\partial \theta_i} - \frac{\partial(\psi_1 + \psi_2)}{\partial r_i}\right] \tag{2.3}$$

(Re is the symbol for the real part).

All functions must be converted to a single system of coordinates in order to use the equations at (2.3). Let us transfer from the r_1, θ_1 system to the r_2, θ_2 system of coordinates to do so

$$r_1^2 = r_2^2 + 2b r_2 \cos\theta_2 + b^2,$$

where b is the distance between the axes of the cylinders.

Using the summation theorem for an arbitrary cylindrical function

$Z_n(\lambda r_1)$

$$Z_n(\lambda r_1)\begin{matrix}\cos n\theta_1\\ \sin n\theta_1\end{matrix} = \sum_{j=0}^{\infty} (-1)^j \varepsilon_j J_j(\lambda r_2)\left[(-1)^j Z_{n-j}(\lambda b) \pm Z_{n+j}(\lambda b)\right]\begin{matrix}\cos j\theta_2\\ \sin j\theta_2\end{matrix}.$$

we obtain the relationships /60

$$\varphi = \sum_{n=0}^{\infty} \left[A_n^{(2)} H_n(\alpha r_2) + J_n(\alpha r_2) \sum_{m=0}^{\infty} A_m^{(1)} \varphi_{nm}\right] \cos n\theta_2;$$

$$\psi = \sum_{n=1}^{\infty} \left[B_n^{(2)} H_n(\beta r_2) + J_n(\beta r_2) \sum_{m=0}^{\infty} B_m^{(1)} \psi_{nm}\right] \sin n\theta_2. \qquad (2.4)$$

Here

$$\varphi_{nm} = (-1)^n \varepsilon_n \left[H_{m+n}(\alpha b) + (-1)^n H_{m-n}(\alpha b)\right];$$

$$\psi_{nm} = (-1)^n \left[-H_{m+n}(\beta b) + (-1)^n H_{m-n}(\beta b)\right];$$

$J_m(w)$ is a Bessel function of the first kind; $\varepsilon_o = 0.5$; $\varepsilon_n = 1$; $n \geq 1$.

Now the equations at (2.3), with Equation (2.4) taken into consideration yield

$$v_r = \mathrm{Re}\,\alpha \left[A_0^{(2)} H_0'(\alpha r_2) + J_0'(\alpha r_2) \sum_{m=0}^{\infty} A_m^{(1)} \varphi_{0m}\right] e^{-i\omega t} +$$

$$+ \mathrm{Re} \sum_{n=1}^{\infty} \cos n\theta_2 \left\{\alpha \left[A_n^{(2)} H_n'(\alpha r_2) + J_n'(\alpha r_2) \sum_{m=0}^{\infty} A_m^{(1)} \varphi_{nm}\right] + \right.$$

$$\left. + \frac{n}{r_2} \left[B_n^{(2)} H_n(\beta r_2) + J_n(\beta r_2) \sum_{m=1}^{\infty} B_m^{(1)} \psi_{nm}\right]\right\} e^{-i\omega t};$$

$$v_\theta = -\mathrm{Re} \sum_{n=1}^{\infty} \sin n\theta_2 \left\{\frac{n}{r_2} \left[A_n^{(2)} H_n(\alpha r_2) + J_n(\alpha r_2) \sum_{m=0}^{\infty} A_m^{(1)} \varphi_{nm}\right] + \right.$$

$$\left. + \beta \left[B_n^{(2)} H_n'(\beta r_2) + J_n'(\beta r_2) \sum_{m=1}^{\infty} B_m^{(1)} \psi_{nm}\right]\right\} e^{-i\omega t}.$$

Taking the condition of symmetry into consideration, we find

$$A_n^{(1)} = (-1)^n A_n^{(2)} = (-1)^n A_n;\ B_n^{(1)} = (-1)^n B_n^{(2)} = (-1)^n B_n. \tag{2.5}$$

Whereupon, having satisfied the boundary conditions at (1.4), we obtain an infinite system of equations on the surface of the second cylinder for finding A_n and B_n

$$\alpha H_n'(\alpha a) A_n + \alpha J_n'(\alpha a) \sum_{m=1}^{\infty} (-1)^m \varphi_{nm} A_m + \frac{n}{a} H_n(\beta a) B_n + \frac{n}{a} J_n(\beta a) \times \quad /61$$

$$\times \sum_{m=1}^{\infty} (-1)^m \psi_{nm} B_m = V_n - \alpha J_n'(\alpha a) \varphi_{n0} A_0;$$

$$\frac{n}{a} H_n(\alpha a) A_n + \frac{n}{a} J_n(\alpha a) \sum_{m=1}^{\infty} (-1)^m \varphi_{nm} A_m + \beta H_n'(\beta a) B_n + \beta J_n'(\beta a) \times$$

$$\times \sum_{m=1}^{\infty} (-1)^m \psi_{nm} B_m = -\frac{n}{a} J_n(\alpha a) \varphi_{n0} A_0;$$

$$A_0 = \left[V_0 - \alpha J_0'(\alpha a) \sum_{m=1}^{\infty} (-1)^m \varphi_{0m} A_m \right] [a H_0'(\alpha a) - J_0'(\alpha a) \varphi_{00}]^{-1}. \tag{2.6}$$

These conditions are automatically fulfilled on the first cylinder in view of Equation (2.5). The solution of this system completes the solution of the problem posed.

#3. Let us prove that the system at (2.6) can be solved by the reduction method. If new unknowns $A_n = x_n J_n(\alpha a)$, $B_n = y_n J_n(\beta a)$ are introduced in place of the unknowns A_n, B_n, the system at (2.6) will be transformed into

$$x_n + \sum_{m=1}^{\infty} a_{nm} x_m + b_n y_n + \sum_{m=1}^{\infty} c_{nm} y_m = f_n;$$

$$y_n + \sum_{m=1}^{\infty} d_{nm} y_m + l_n x_n + \sum_{m=1}^{\infty} k_{nm} x_m = F_n \quad (n = 1, 2, \ldots). \tag{3.1}$$

The following notations are introduced

$$a_{nm} = \frac{(-1)^m J_m(\alpha a)}{H_a(\alpha a)} \Phi_{nm};\quad k_{nm} = \frac{(-1)^m \alpha a J_n'(\alpha a) J_m(\alpha a)}{n H_n(\beta a) J_n(\beta a)} \Phi_{nm};$$

$$c_{nm} = \frac{(-1)^m \beta a J_n'(\beta a) J_m(\beta a) \psi_{nm}}{n H_n(\alpha a) J_n(\alpha a)};\quad d_{nm} = \frac{(-1)^m J_m(\beta a) \psi_{nm}}{H_n(\beta a)};$$

$$b_n = \frac{\beta a H_n'(\beta a) J_n(\beta a)}{n H_n(\alpha a) J_n(\alpha a)};\quad l_n = \frac{\alpha a H_n'(\alpha a) J_n(\alpha a)}{n H_n(\beta a) J_n(\beta a)};$$

$$f_n = \frac{V_0 \varphi_{n0} J_n'(\alpha a)}{\alpha H_n(\alpha a) J_n(\alpha a) [H_0'(\alpha a) + \varphi_{00} J_0'(\alpha a)]};$$

$$F_n = \frac{a}{n H_n(\beta a) J_n(\alpha a)} \left[V_a - \frac{V_0 \varphi_{n0} J_n'(\alpha a)}{H_0'(\alpha a) + \varphi_{00} J_0'(\alpha a)} \right];$$

$$\Phi_{nm} = \varphi_{nm} - \frac{\varphi_{n0} \varphi_{0m} J_0'(\alpha a)}{H_0'(\alpha a) + \varphi_{00} J_0'(\alpha a)}.$$

(3.2)

Let us transfer the terms with unknowns y_n in the first group of the equations at (3.1) to the right side, and let us investigate the system thus obtained

$$x_n + \sum_{m=1}^{\infty} a_{nm} x_m = f_n^*;\quad f_n^* = f_n - b_n y_n - \sum_{m=1}^{\infty} c_{nm} y_m.$$

(3.3)

Let us estimate the factors a_{nm} and the right sides f_n^* for large numbers n. Without pausing to dwell on the methodology used to find these estimations [1], and omitting the intermediate calculations, let us write

$$|a_{nm}| < K_1 \frac{(m+n)!}{m!\, n!} \left[1 + K_2 \frac{m!\, n!}{(m+n)!} \right] \left(\frac{a}{b} \right)^{m+n};$$

$$|c_{nm}| < K_3 \frac{(m+n)!}{m!\, n!} \left(\frac{a}{b} \right)^{m+n};\quad |b_n| < K_4;$$

$$|f_n| < K_5 n \left(\frac{a}{b} \right)^n \quad (K_i = \text{const} > 0).$$

(3.4)

Let us sum the right sides of the inequality for $|a_{nm}|$ at a time

$$\sum_{m=1}^{\infty} |a_{nm}| < K_6 \left(\frac{a}{b-a}\right)^n. \qquad /62$$

Since $2a < b$, it is always possible to find that number N, beginning with which the condition

$$\sum_{m=1}^{\infty} |a_{Nm}| < 1.$$

is satisfied.

Moreover, if it is considered that $B_n H_n(\alpha a)$ are the coefficients of the expansion of a Fourier function that has continuous derivatives of at least second order, and that, as a result, $y_n \sim n^{-2}$, we obtain the estimate for f_n^* as

$$|f_n^*| < K_7 \left(1 - \sum_{m=N+1}^{\infty} |a_{nm}|\right).$$

The infinite system of equations at (3.3) is quasi-regular. Solving it, we find the constants x_n as a function of y_m. Substituting $x_n = x_n(y_m)$ in the second group of equations at (3.1) to find y_m, we obtain an infinite system which too is quasi-regular.

#4. Let us determine the distance between the cylinders at which the radiation from one has no practical effect on the radiation from the other. It is sufficient to limit the finding to the case of n = 0 to obtain qualitative estimates. The function φ in this case is in the form

$$\varphi = \frac{V_0}{\alpha} \frac{H_0(\alpha r_2) + J_0(\alpha r_2) H_0(\alpha b)}{H_0'(\alpha a) + J_0'(\alpha a) H_0(\alpha b)} e^{-i\omega t}. \qquad (4.1)$$

Velocity and pressure are given by the expressions

$$v_r = V_0 \operatorname{Re} \frac{H_1(\alpha r_2) + J_1(\alpha r_2) H_0(\alpha b)}{H_1(\alpha a) + J_1(\alpha a) H_0(\alpha b)} e^{-i\omega t};$$

$$p = -\rho_0 V_0 \operatorname{Re} \frac{i\omega - \left(\nu' + \frac{4}{3}\nu\right)\alpha^2}{\alpha} \frac{H_0(\alpha r_2) + J_0(\alpha r_2) H_0(\alpha b)}{H_1(\alpha a) + J_1(\alpha a) H_0(\alpha b)} e^{-i\omega t} \qquad (4.2)$$

Let us investigate the effect of the adjacent cylinder on the pressure distribution. Let us introduce the Strouhal number (for the cited frequency) k, and the Reynolds number R. Let us present the formula for the pressure in the form

$$p = -\varrho_0 c V_0 \operatorname{Re} i \left(\frac{1 + ikR^{-1}}{1 + k^2 R^{-2}} \right)^{\frac{1}{2}} \frac{H_0(\alpha r_2)}{H_1(\alpha a)} (1 + \delta) e^{-i\omega t},$$

where the magnitude δ, when $r_2 = a$, is equal to

$$\delta = -\frac{2iH_0(\alpha b)}{\pi \alpha a H_0(\alpha a)[H_1(\alpha a) + J_1(\alpha a) H_0(\alpha b)]} \tag{4.3}$$

and characterizes the effect of the adjacent cylinder. From whence it will be seen that $|\delta| \to 0$, when $b \to \infty$. /63

Let us consider the two limit cases, one when the R number for the medium is large, and one when it is small. Let us suppose we have the inequalities $R \gg 1$, $k \gg 1$, $R \gg k$. In that case $\alpha a \approx k(1 + ik/2k)$. Using asymptotic representations for cylindrical functions, we obtain

$$\delta = \sqrt{\frac{R}{\pi k b}} e^{-\frac{k\lambda}{2R}} \left\{ \left(1 + \frac{k}{4R}\right) \cos\lambda - \left(1 - \frac{k}{4R}\right) \sin\lambda + \right.$$
$$\left. + i \left[\left(1 + \frac{k}{4R}\right) \sin\lambda + \left(1 - \frac{k}{4R}\right) \cos\lambda \right] \right\}, \quad \lambda = \left(\frac{b}{a} - 2\right) k. \tag{4.4}$$

The modulus of δ is extremely small when $b > 2a$ for the medium with fading to low viscosity, and this is indicative of the weak interference between the cylinders.

Figure 2 shows the change in δ in the region of small kR^{-1}. As we see, an increase in the viscosity reduces that minimal distance between the cylinders at which their interference with each other is not yet perceived. This distance is shorter the higher the frequency of the oscillations.

The inequality $R \ll k$ corresponds to the second limit case, and for the magnitude δ, characterizing the effect the cylinders have on each other, we obtain

$$\delta = -\frac{2\sqrt{i} H_0\left(\frac{b}{a}\sqrt{ikR}\right)}{\pi \sqrt{kR}\, H_0(\sqrt{ikR})[H_1\sqrt{ikR}) + J_1\sqrt{ikR}) H_0\left(\frac{b}{a}\sqrt{ikR}\right)}.$$

This formula can be represented in a form more convenient for computation as

$$\delta = \frac{\sqrt{2}}{\pi\varkappa(\Delta_1^2+\Delta_2^2)}\left\{(\Delta_1+\Delta_2)\,\mathrm{her}\,\frac{\varkappa b}{a}+(\Delta_1-\Delta_2)\,\mathrm{hei}\,\frac{\varkappa b}{a}+\right.$$

$$\left.+\,i\left[(\Delta_1-\Delta_2)\,\mathrm{her}\,\frac{\varkappa b}{a}-(\Delta_1+\Delta_2)\,\mathrm{hei}\,\frac{\varkappa b}{a}\right]\right\}\quad(\varkappa=\sqrt{kR}).$$

Here

$$\Delta_1 = -\,\mathrm{her}\,\varkappa\,\mathrm{her}_1\varkappa+\mathrm{hei}\,\varkappa\,\mathrm{hei}_1\varkappa+\mathrm{her}\,\frac{\varkappa b}{a}\,(-\,\mathrm{her}\,\varkappa\,\mathrm{ber}_1\varkappa+\mathrm{hei}\,\varkappa\,\mathrm{bei}_1\varkappa)+$$

$$+\,\mathrm{hei}\,\frac{\varkappa b}{a}\,(\mathrm{her}\,\varkappa\,\mathrm{bei}_1\varkappa+\mathrm{hei}\,\varkappa\,\mathrm{ber}_1\varkappa);$$

$$\Delta_2 = \mathrm{hei}\,\varkappa\,\mathrm{her}_1\varkappa+\mathrm{her}\,\varkappa\,\mathrm{hei}_1\varkappa+\mathrm{her}\,\frac{\varkappa b}{a}\,(\mathrm{hei}\,\varkappa\,\mathrm{ber}_1\varkappa+\mathrm{her}\,\varkappa\,\mathrm{bei}_1\varkappa)+$$

$$+\,\mathrm{hei}\,\frac{\varkappa b}{a}\,(-\,\mathrm{hei}\,\varkappa\,\mathrm{bei}_1\varkappa+\mathrm{her}\,\varkappa\,\mathrm{ber}_1\varkappa),$$

while her $\varkappa$, ber $\varkappa$, and hei $\varkappa$, bei $\varkappa$ are the real and imaginary parts of the Hankel and Bessel functions, respectively

$$H_n^{(1)}(\varkappa\sqrt{i}) = (-1)^{n+1}(\mathrm{her}_n\varkappa - i\,\mathrm{hei}_n\varkappa);$$

$$J_n(\varkappa\sqrt{i}) = (-1)^n(\mathrm{ber}_n\varkappa - i\,\mathrm{bei}_n\varkappa)\quad(\varkappa>0).$$

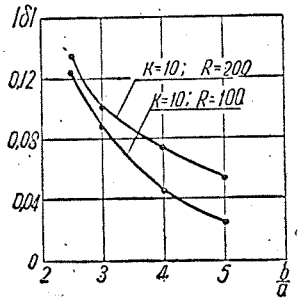

Figure 2

Determining the modulus of the magnitude δ, we have /64

$$|\delta| = \frac{2}{\pi\varkappa}\left(\frac{\mathrm{her}^2\frac{\varkappa b}{a}+\mathrm{hei}^2\frac{\varkappa b}{a}}{\Delta_1^2+\Delta_2^2}\right)^{\frac{1}{2}}. \qquad (4.5)$$

In order to find the regions in which $|\delta| \lesssim \gamma$ (γ is a small number limiting the error in the consideration) we must find the solution of the equation

$$\mathrm{her}^2\frac{\varkappa b}{a} + \mathrm{hei}^2\frac{\varkappa b}{a} = \frac{\pi^2\gamma^2}{4}(\Delta_1^2 + \Delta_2^2). \tag{4.6}$$

If we put $\gamma = 0.1$, we can use the roots of this equation to construct the graphic of the dependence of the b/a ratio on the magnitude of $\varkappa = \sqrt{kR}$ (fig. 3). As will be seen from this graphic, when the R number is

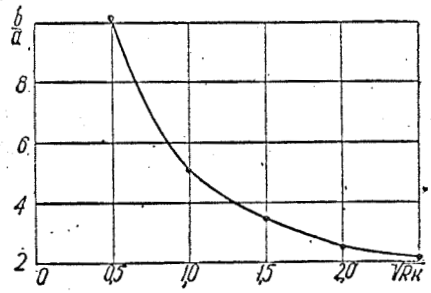

Figure 3

very small the interference between the cylinders will be much greater than it will be in the case of too large an R number. Moreover, with reduction in the R number there is an increase in the minimal distance at which the pressure distribution near the cylinders will not depend on their interference with each other. As in the region of low viscosities, long-wave radiation is quenched more quickly than short-wave.

In conclusion, let us note that in the second limit case no limitation was imposed on the parameter $\varkappa$. However,, when $\varkappa \gg 1$, the interference between the cylinders is very slight, and the magnitude δ has the form

$$|\delta| \approx \sqrt{\frac{2a}{\pi\varkappa b}}\exp\left[-\sqrt{\frac{\varkappa}{2}}\left(\frac{b}{a} - 2 + \frac{\pi}{4}\right)\right].$$

If $b = 2a$ and $\varkappa = 1$, $|\delta| \approx 0.3$, and the modulus of the magnitude δ rapidly decreases with increase in $\varkappa$.

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